Voluntary Carbon Market in the Agriculture Sector: A Game-Theoretic and Mechanism Design Approach

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Abstract

Small and marginal farmers in India encounter significant barriers in accessing emerging carbon markets due to high transaction costs, limited bargaining power, and resource constraints. In response, the Government of India has proposed a voluntary carbon market (VCM) for the agriculture sector to incentivize sustainable practices and support climate goals.However, fair pricing, equitable revenue distribution, and effective participation mechanisms remain critical challenges. This study addresses these issues using a dual approach.
(1) Cooperative Game Theory Approach: we apply cooperative game theory to examine coalition formation among farmers—primarily through Farmer Producer Organizations—assessing the benefits of aggregation, stability of coalitions via the Core concept, and the fairness of revenue distribution through the Shapley value.
(2) Mechanism Design Approach: we investigate mechanism design for carbon credit pricing and trading by comparing the performance of Shapley-based allocation with traditional auction mechanisms such as the Vickrey-Clarke-Groves (VCG) auction.

Simulation results on synthetic yet representative data indicate that farmer coalitions can substantially enhance individual payoffs, with the Shapley allocation consistently ensuring equitable outcomes and satisfying individual rationality—unlike conventional auction methods. These findings underscore the importance of careful parameter calibration and offer actionable insights for designing an efficient and equitable VCM tailored to the needs of Indian agriculture.

The implementations of the models, simulations, and analysis discussed in this study are available in the GitHub repository. [1]

1 Introduction

Agriculture forms the backbone of India's economy, contributing significantly to the GDP (nearly 18%) and supporting the livelihoods of over half its population[17, 18]. The sector is vital not only for national food security but also for sustaining rural economies and social well-being. However, Indian agriculture faces mounting pressures from rising input costs, land tenure complexities[16], and, critically, the impacts of climate change[26]. Increased frequency of extreme weather events, coupled with the sector's own contribution to greenhouse gas (GHG) emissions[18], This brings two main challenges: making farming stronger against problems and reducing harm to the environment. India has committed to ambitious climate targets, including significant carbon sequestration goals under its Nationally Determined Contribution (NDC)[19]. Therefore, embracing sustainable agricultural methods is critical for long-term viability.

[1] GitHub Repo : `https://github.com/Mahanth-Maha/GameTheory2025MiniProject`

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Recognizing these challenges, the Government of India has taken steps to promote sustainability within the sector. A key recent initiative is the proposed framework for a Voluntary Carbon Market (VCM) in agriculture, detailed in the "Framework for Voluntary Carbon Market In Agriculture Sector (2024)" by the Ministry of Agriculture and Farmers Welfare[17]. The Voluntary Carbon Market (VCM) aims to incentivize farmers to adopt environmentally sustainable practices, such as agroforestry, conservation tillage, improved water management, reduced chemical fertilizer use, and enhancing soil organic carbon (SOC)—by enabling them to generate and sell carbon credits[17]. This mechanism leverages the potential for carbon sequestration and emission reduction within agriculture, potentially contributing significantly to national climate goals and generating substantial economic value[14]. It creates a potential new revenue stream, particularly crucial for India's agricultural landscape, which is dominated by small (less than 2 hectares) and marginal farmers who cultivate over 86% of the vast agricultural land [approx. 60% of total land area, 17, 31].

While the VCM framework offers a promising pathway, its successful implementation hinges on overcoming significant hurdles, especially for smallholder farmers. These farmers often face high transaction costs associated with monitoring, reporting, and verification (MRV) required for carbon credit generation, possess limited technical knowledge and resources for adopting new practices, and lack bargaining power when dealing individually with credit buyers or project developers [6, 17, 2]. The government framework acknowledges these issues and promotes collective action through Farmer Producer Organizations (FPOs) or similar community-based bodies to aggregate farmers, reduce costs, and facilitate market access [17, 39]. However, critical questions remain regarding how the collective benefits generated by these FPOs should be fairly distributed among participating farmers and how the market for trading these credits should be structured to ensure fairness, efficiency, and robust participation.

This paper addresses these critical questions by applying analytical tools from game theory and mechanism design to the proposed Indian agricultural VCM. Our work focuses on two primary objectives aligned with the challenges identified:

(1) **Analyzing Coalition Formation and Fair Allocation in FPOs:** Using cooperative game theory, we model FPOs as coalitions of farmers working together to generate carbon credits. We aim to understand the conditions under which such coalitions are stable (i.e., the framework ensures that farmers have a continued incentive to stay within the FPO.) using the concept of the *Core*[10]. Furthermore, we investigate methods for fair and equitable distribution of the collective revenue generated from carbon credit sales among FPO members, employing the *Shapley value*[29, 35] as a key solution concept that allocates value based on marginal contributions. We model the potential synergy or economies of scale within the coalition using a characteristic value function.

(2) **Designing and Evaluating Market Mechanisms for Carbon Credit Trading:** We utilize mechanism design principles[23, 21] to explore rules for pricing and trading the agricultural carbon credits generated. The goal is to identify mechanisms that are *incentive-compatible* (encouraging truthful reporting of costs or values), transparent, and fair, particularly considering the heterogeneity among farmers[37]. We specifically simulate and compare the performance of allocation mechanisms derived from cooperative game theory (like Shapley value-based distribution) against standard auction formats like the Vickrey-Clarke-Groves (VCG) auction[15, 4, 36], evaluating them based on metrics like farmer profitability, fairness (Gini coefficient), and individual rationality (IR).

By simulating farmer interactions within coalitions and evaluating different market structures, this study provides quantitative insights into the potential outcomes of the proposed VCM. We analyze the impact of key parameters, such as the degree of synergy within FPOs and the design of the trading mechanism, on farmer participation, profitability, and the overall equity of the market. The findings aim to inform policymakers and stakeholders on structuring an effective VCM that truly benefits India's small and marginal farmers while contributing to national climate and sustainability objectives.

2 Literature Review

The development of a Voluntary Carbon Market (VCM) for the Indian agriculture sector intersects several fields of study, primarily environmental economics, cooperative game theory, and mechanism design. This section reviews key literature relevant to the challenges and approaches discussed in this paper, particularly focusing on farmer cooperation and market design for carbon credits generated by smallholders.

2.1 Voluntary Carbon Markets and Agricultural Challenges

The concept of using market-based mechanisms to address climate change has evolved significantly since the initial frameworks like the Kyoto Protocol. While compliance markets exist, voluntary carbon markets (VCMs) have gained prominence, driven by corporate net-zero commitments and individual desire to offset emissions [26]. The Paris Agreement's Article 6 further provides mechanisms for international cooperation and market-based approaches [17].

Applying VCM principles to agriculture, however, presents distinct challenges [37]. Unlike industrial point sources, agricultural emissions and sequestration are dispersed across numerous, often small, landholdings. Quantifying carbon benefits from practices like improved soil management or agroforestry involves biological complexities and uncertainties, demanding robust Monitoring, Reporting, and Verification (MRV) protocols [7]. Transaction costs associated with MRV and project development can be prohibitively high for individual smallholder farmers, who dominate the Indian agricultural landscape [6, 16]. Furthermore, issues of land tenure (leased vs. owned land), information asymmetry, limited access to finance for adopting sustainable practices, and ensuring 'additionality' (proving credits represent activity beyond business-as-usual) are critical hurdles [17]. The Indian government's framework explicitly acknowledges these difficulties and highlights the need for aggregation and capacity building [17]. National initiatives promoting sustainable methods like Natural Farming also align with the goals of agricultural VCMs [24, 9].

2.2 Cooperative Game Theory for Farmer Aggregation

Given the necessity of aggregation to overcome transaction costs and enhance bargaining power, Farmer Producer Organizations (FPOs) are positioned as key enablers in the Indian agricultural VCM [17]. Cooperative game theory provides a helpful framework to examine how farmer coalitions form, remain stable, and function internally [10].

Modeling Coalition Value: The potential benefits derived by an FPO (coalition S) depend on the collective actions of its members (N). This is captured by the characteristic function $v(S)$, representing the total value (e.g., net revenue from carbon credits) the coalition S can generate. Modeling this function realistically is crucial. Approaches often consider additive components (sum of individual contributions) and non-additive components representing synergies or costs. For instance, [39] model FPO benefits for biopesticide adoption, while [1, 20] analyze environmental games with spatial externalities where cooperation yields supra-additive benefits. Our work employs a functional form

$$v(S) = \alpha \sum_{i \in S} r_i + \beta (\sum_{i \in S} r_i)^2$$

where:

- r_i is the baseline payoff for farmer i,
- α scales the baseline contribution, and
- β models the quadratic synergy/economy of scale effect.

The choice and estimation of α and β significantly influence the predicted outcomes, as shown in our experiments.

Coalition Stability: The Core, A fundamental question is whether an FPO can maintain its membership. If a subgroup of farmers can achieve a better collective outcome by leaving the FPO and forming their own smaller coalition, the FPO is considered unstable.

The *Core* [12, 28] formalizes this notion, defining the set of payoff allocations $(x_1, ..., x_n)$ such that no subgroup $S \subset N$ can obtain more than $\sum_{i \in S} x_i$ by acting alone (i.e., $\sum_{i \in S} x_i \geq v(S)$ for all S), while ensuring the total value $v(N)$ is distributed. A non-empty Core guarantees the existence of at least one stable allocation. The computational complexity of finding the Core can be high, and it may be empty in some games. The concept of *approximate stability*, where deviations are only triggered if the gain exceeds a certain threshold or proportion, offers a relaxation and guarantees existence more broadly [11].

Our experimental analysis checks for Core stability for small FPO sizes under different allocation rules, as the time complexity of exactly computing the Core is exponential in the number of players [10].

Fair Allocation: The Shapley Value, Assuming an FPO generates a collective benefit $v(N)$, the question arises of how to distribute this value fairly among its members.

The *Shapley value* [29] gives a fairness rule based on clear principles such as efficiency, treating similar players equally, and giving nothing to those who contribute nothing. It assigns each farmer their expected marginal contribution over all possible orders in which they could join the coalition. The Shapley value has found applications in diverse cooperative settings, including fairly allocating costs or profits in supply chains [35, 38] and designing incentives within FPOs [39].

Our study evaluates the Shapley value as a primary candidate for revenue sharing within the agricultural VCM, assessing its fairness (via Gini coefficient) and its ability to satisfy individual rationality (IR), ensuring farmers are better off joining the FPO than remaining independent.

2.3 Mechanism Design for Carbon Credit Trading

Beyond the internal FPO dynamics, the design of the market where carbon credits are traded is crucial. **Mechanism design** [23, 22] provides the tools to engineer market rules that align individual incentives with desired system outcomes like efficiency and fairness.

Auction Mechanisms: Auctions are common mechanisms for selling goods, including environmental credits [13, 15]. Different auction formats exist, each with distinct properties:

- **VCG Auction:** The Vickrey-Clarke-Groves mechanism [23] is known for achieving efficient allocation (maximizing total surplus) and being strategy-proof (truth-telling is optimal) under certain assumptions (e.g., quasi-linear utilities). It sets payments by looking at how much each bidder's presence affects the others. However, VCG can suffer from potential budget deficits (in general settings, though often not in single-item or simple settings like ours) and computational complexity. Its performance in carbon markets has been studied experimentally [4] and analytically [36].

  Our simulations implement a VCG auction where farmers bid their costs (implicitly, via their willingness to supply at a given price) and evaluate its outcomes.
- **Other Formats:** Uniform-price (where all winners pay the same market-clearing price) and discriminatory-price auctions are also used [4, 36]. These may be simpler but often lack the strong incentive properties like VCG.

  Our experiments include a basic Uniform Price auction for comparison, primarily focusing on VCG as a theoretical benchmark for efficiency.

Key Design Considerations: When designing mechanisms for agricultural carbon credits, several factors are paramount:

- **Heterogeneity:** Farmers vary greatly in the cost of adopting climate-friendly practices—such as improving soil health, shifting to low-emission inputs, or planting trees. These differences are further shaped by variations in farm size, individual attitudes toward risk, and the level of access they have to technical knowledge and market information [37]. The mechanism must handle this heterogeneity effectively.
- **Information Asymmetry:** Farmers typically have better information about their costs and efforts than buyers or regulators. Incentive compatibility is vital to uncover the truthful information.
- **Individual Rationality (Participation Constraint):** As the market is voluntary, the mechanism must ensure farmers receive payoffs that meet or exceed their opportunity cost (their

payoff from traditional farming or not participating) [21]. This condition is more demanding than merely compensating for implementation costs and is a key aspect of our evaluation framework.

- **Fairness:** Beyond efficiency, the distribution of surplus between farmers, buyers, and any intermediary (like an FPO or auctioneer) is a key concern for equity and long-term acceptance [37].

Recent work explores adaptive mechanisms [34] and automated mechanism design [27] to tackle complex market environments, although direct application to the specific Indian agricultural VCM requires further study.

2.4 Positioning of this Work

This study integrates these strands of literature to analyze the specific context of the proposed Indian agricultural VCM. While previous work has examined FPO incentives [39], carbon auction design [4, 36], and mechanism design for heterogeneous agricultural producers [37] separately, our contribution lies in:

1. Explicitly modeling FPOs using cooperative game theory within the VCM framework and evaluating the stability (Core) and fairness (Shapley value) of farmer participation under varying synergy assumptions (α, β).
2. Directly comparing the outcomes (profitability, fairness, IR vs. standalone payoff) of this cooperative game-theoretic allocation approach (Shapley) against a standard market mechanism (VCG auction) using consistent simulations.
3. Providing quantitative evidence on the suitability of different approaches to address the dual goals of farmer welfare and VCM viability in the Indian smallholder context, thereby informing the practical implementation of the government's framework [17].

This work aims to integrate cooperative game theory and mechanism design to analyze the effectiveness of voluntary carbon markets for smallholder farmers. By linking internal dynamics within FPOs to external market mechanisms, the study provides a unified framework to assess both fair revenue sharing and efficient credit pricing—offering insights that support better policy design and farmer participation.

3 Models and Methods

This section formally defines the game-theoretic and mechanism design models employed to analyze coalition formation and carbon credit trading within the proposed Indian agricultural Voluntary Carbon Market (VCM).

3.1 Problem Formulation: The VCM Game

We model the VCM scenario as a game involving multiple agents.

- **Players:** The primary players are the individual farmers, denoted by the set $N = \{1, 2, \ldots, n\}$.
- **Actions/Strategies:** A farmer `i`'s fundamental choice is whether to participate in the VCM, potentially by joining a coalition (FPO), or to continue with standalone farming. If participating, their actions involve adopting specific sustainable agricultural practices which help in generate carbon credits.
- **Payoffs:** The payoff for farmer `i`, denoted by $u_i(\cdot)$, represents their net benefit (e.g., in INR). This payoff depends on their chosen strategy, the actions of other farmers (especially within a coalition), and the market outcomes (e.g., carbon credit price, revenue share). For standalone farming, farmer `i` receives a baseline payoff r_i, derived from their traditional agricultural activities. We assume r_i is known or estimated for each farmer (e.g., from the 'Standalone_Payoff_INR' in our dataset generated from the probability distribution).

- **Information:** We generally assume farmers know their own baseline payoff r_i and potentially their costs for adopting sustainable practices. Information about others payoffs, costs, or the true market value of credits may be unknown or private. (In the auction setting, we assume farmers know their own costs but not those of others.)

3.2 Cooperative Game Model for FPO Coalitions

To analyze the benefits and stability of farmers cooperating through FPOs, we use a cooperative game framework with transferable utility (TU).

Definition 3.1 (Cooperative Game)**.** *A cooperative TU game is defined by a pair (N, v), where $N = \{1, \ldots, n\}$ is the set of players (farmers) and $v : 2^N \to \mathbb{R}$ is the characteristic function. For any coalition $S \subseteq N$, $v(S)$ represents the total payoff (value) that the members of coalition S can jointly achieve by cooperating, with $v(\emptyset) = 0$.*

Characteristic Function ($v(S)$). The definition of $v(S)$ is crucial as it encodes the value generated by cooperation. Based on our experimental setup, we model the value generated by a farmer coalition S (representing an FPO) as:

$$v(S) = \alpha \sum_{i \in S} r_i + \beta \left(\sum_{i \in S} r_i \right)^2 \tag{1}$$

where:

- $r_i \geq 0$ is the standalone payoff (baseline income) of farmer `i`.
- $\alpha \geq 0$ is a parameter scaling the linear contribution of individual baseline payoffs. If $\alpha = 1$, the coalition guarantees at least the sum of individual payoffs before considering synergy. $\alpha > 1$ could represent inherent efficiency gains even without synergy, while $\alpha < 1$ might represent baseline coordination costs.
- $\beta \geq 0$ is a parameter capturing the synergistic effects of scale within the coalition.

This function assumes that the value generated is primarily related to the members' baseline capabilities (r_i) and the potential for synergy (β). It simplifies the complex reality of carbon credit generation and costs but allows for tractable analysis of cooperation incentives. The game defined by Equation 1 is superadditive if $\alpha \geq 1$ and $\beta \geq 0$, meaning $v(S \cup T) \geq v(S) + v(T)$ for disjoint S, T, which generally incentivizes the formation of larger coalitions [25].

Stability: The Core, As defined in Section 2, the Core identifies stable payoff allocations. An allocation $x = (x_1, \ldots, x_n)$ is in the Core of the game (N, v) if $\sum_{i \in N} x_i = v(N)$ and $\sum_{i \in S} x_i \geq v(S)$ for all $S \subseteq N$. A non-empty Core is desirable for the long-term viability of the FPO.

Theorem 3.1 (Bondareva-Shapley Theorem [25])**.** *A cooperative TU game (N, v) has a non-empty Core if and only if it is balanced. A game is balanced if for every balanced collection of coalitions $\mathcal{B}$ (where weights $\lambda_S > 0$ exist for $S \in \mathcal{B}$ such that $\sum_{S \in \mathcal{B}: i \in S} \lambda_S = 1$ for all $i \in N$), it holds that $\sum_{S \in \mathcal{B}} \lambda_S v(S) \leq v(N)$.*

While checking balance directly is complex, games that are *convex* always have a non-empty Core [29]. A game is convex if $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$ for all $S \subseteq T \subseteq N \setminus \{i\}$. Our specific characteristic function (Eq. 1) with $\beta > 0$ is generally convex, suggesting the Core should be non-empty. Algorithm 1 outlines the procedure used in our simulations to check if a given payoff vector lies within the Core. This involves verifying efficiency and checking the coalition rationality constraint for all $2^n - 2$ non-trivial, proper subsets $S \subset N$.

The exponential complexity of iterating through all subsets in Algorithm 1 limits its practical application to small n (typically $n \leq 15$), as reflected in our experimental parameters ('CORE_CHECK_THRESHOLD').

Fair Allocation: The Shapley Value offers a unique allocation of value among participants, derived logically from a set of fairness-based axioms. [29].

Theorem 3.2 (Shapley Axioms [29, 22])**.** *The Shapley value $\phi(v) = (\phi_1(v), \ldots, \phi_n(v))$ is the unique allocation rule satisfying the following three axioms:*

Algorithm 1 Check if Payoff Vector x is in the Core

Input: Payoff vector $x = (x_1, \dots, x_n)$, Characteristic function values $v(S)$ for all $S \subseteq N$, Tolerance $\epsilon > 0$.

Output: Boolean 'True' if x is in the Core, 'False' otherwise.

Compute total allocated payoff $X_{total} = \sum_{i \in N} x_i$.

Compute grand coalition value $V_N = v(N)$.

if $|X_{total} - V_N| > \epsilon$ **then** ▷ Check Efficiency

  return 'False'

end if

for each coalition $S \subseteq N$, where $S \neq \emptyset$ and $S \neq N$ **do**

  Compute coalition allocation $X_S = \sum_{i \in S} x_i$.

  Get coalition value $V_S = v(S)$.

  if $X_S < V_S - \epsilon$ **then** ▷ Check Coalition Rationality

    return 'False' ▷ S is a blocking coalition

  end if

end for

return 'True' ▷ No blocking coalitions found

1. ***Symmetry:*** *For any permutation π on the set of players N and any player $i \in N$, we have*
$$\phi_{\pi(i)}(v^\pi) = \phi_i(v),$$
where $v^\pi(S) = v(\pi^{-1}(S))$ is the permuted value function. This axiom implies that only the role of a player in the game matters, not their labels.

2. ***Linearity:*** *For any two coalitional games v_1, v_2 and any scalar $p \in [0, 1]$, define $v = pv_1 + (1-p)v_2$ by*
$$v(S) = pv_1(S) + (1-p)v_2(S), \quad \forall S \subseteq N.$$
Then, for every player $i \in N$,
$$\phi_i(v) = p\phi_i(v_1) + (1-p)\phi_i(v_2).$$
This axiom ensures that the Shapley value is a linear operator over games.

3. ***Carrier (or Null Player):*** *Let $D \subseteq N$ be a carrier for the game (N, v), i.e., $v(S) = v(S \cap D)$ for all $S \subseteq N$. Then:*
$$\phi_i(v) = 0 \quad \textit{for all } i \notin D, \quad \textit{and} \quad \sum_{i \in D} \phi_i(v) = v(N).$$
This axiom ensures that only players in the carrier set (influential players) receive a non-zero allocation, and they divide the total surplus among themselves.

Theorem 3.3 (Shapley's Uniqueness Theorem [29, 22])**.** *With the above three axioms in place, we can now state the celebrated result due to Shapley:*

There exists a unique value mapping $\phi : \mathbb{R}^{2^n-1} \to \mathbb{R}^n$ that satisfies the above axioms. This mapping is given by:
$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(n - |S| - 1)!}{n!} \left[v(S \cup \{i\}) - v(S)\right], \quad \forall i \in N.$$

Calculating the Shapley value involves computing the marginal contribution of each player `i` to every possible coalition S not containing `i`, weighted by the probability of that coalition forming just before `i` joins in a random permutation of players.

Exact Calculation: For small n (e.g., $n \leq 10 - 12$), the Shapley value can be computed exactly by iterating through all $n!$ permutations of players (Algorithm 2) or by iterating through all 2^{n-1} coalitions S for each player `i` using the Shapley formula as per Theorem 3.3.

Approximate Calculation: For larger n, where $n!$ or 2^n becomes computationally intractable, the Shapley value is typically approximated using Monte Carlo sampling (Algorithm 3). This involves generating a large number (M) of random permutations and averaging the marginal contributions observed for each player. The accuracy of the approximation improves with M. Our experiments use this method for $n > 10$.

Algorithm 2 Exact Shapley Value Calculation (Permutation Method)

Input: Set of players N, Characteristic function v.
Output: Shapley value vector $\phi = (\phi_1, \ldots, \phi_n)$.

Initialize $\phi_i = 0$ for all $i \in N$.
Let Π be the set of all $n!$ permutations of N.
for each permutation $\pi = (\pi(1), \ldots, \pi(n)) \in \Pi$ **do**
  for $k = 1$ to n **do**
    Let $i = \pi(k)$.
    Let $S_{k-1} = \{\pi(1), \ldots, \pi(k-1)\}$ (with $S_0 = \emptyset$).
    Compute marginal contribution $MC_i = v(S_{k-1} \cup \{i\}) - v(S_{k-1})$.
    $\phi_i \leftarrow \phi_i + MC_i$.
  end for
end for
for $i = 1$ to n **do**
  $\phi_i \leftarrow \phi_i / n!$.
end for
return ϕ.

Algorithm 3 Approximate Shapley Value Calculation (Monte Carlo)

Input: Set of players N, Characteristic function v, Number of samples M.
Output: Approximate Shapley value vector $\hat{\phi} = (\hat{\phi}_1, \ldots, \hat{\phi}_n)$.

Initialize $\hat{\phi}_i = 0$ for all $i \in N$.
for $m = 1$ to M **do**
  Generate a random permutation $\pi = (\pi(1), \ldots, \pi(n))$ of N.
  for $k = 1$ to n **do**
    Let $i = \pi(k)$.
    Let $S_{k-1} = \{\pi(1), \ldots, \pi(k-1)\}$ (with $S_0 = \emptyset$).
    Compute marginal contribution $MC_i = v(S_{k-1} \cup \{i\}) - v(S_{k-1})$.
    $\hat{\phi}_i \leftarrow \hat{\phi}_i + MC_i$.
  end for
end for
for $i = 1$ to n **do**
  $\hat{\phi}_i \leftarrow \hat{\phi}_i / M$.
end for
return $\hat{\phi}$.

3.3 Mechanism Design: VCG Auction for Carbon Credits

To model the trading aspect where farmers (or FPOs) sell carbon credits to buyers, we simulate a VCG auction [33, 3, 8, 23].

Setting:

- Sellers: A set of farmers/FPOs $N = \{1, \ldots, n\}$, where each seller `i` offers a quantity q_i (Potential_Carbon_Credits_tCO2e) of credits and has a private cost c_i per credit (True_Cost_per_Credit_INR).
- Buyer(s): We simplify to a single representative buyer (or a fixed market demand price) willing to pay up to p_{max} per credit. In other simulations, we model a buyer demanding a total quantity Q_{demand}.
- Outcome: The mechanism determines which sellers $W \subseteq N$ sell their credits (allocation) and the payment P_i each winning seller $i \in W$ receives.

VCG Mechanism Rules: Assuming sellers bid their true cost per credit c_i (incentivized by the mechanism's properties):

1. **Allocation Rule:** Select the set of sellers W^* that maximizes the total social surplus, given the buyer's maximum price p_{max}. This means selecting all sellers `i` such that their cost c_i is less than or equal to p_{max}.
$$W^* = \{i \in N \mid c_i \leq p_{max}\}$$
The total surplus generated is $\sum_{i \in W^*} q_i(p_{max} - c_i)$.
2. **Payment Rule (Clarke Pivot):** Each winning seller $i \in W^*$ receives a payment P_i equal to the hypothetical value achieved by others if `i` had not participated, minus the actual value achieved by others with `i`'s participation, plus their own cost. In a simple setting with a price threshold p_{max}, this often simplifies. Let $c_{critical}$ be the lowest cost among the sellers not selected (or p_{max} if all sellers with $c_i \leq p_{max}$ are selected). A common implementation (reflected in our VCG code for a price threshold) sets the payment per credit for winner `i` to this critical cost($c_{critical}$).
$$P_i = q_i \times c_{critical}$$
where $c_{critical} = \min(\{c_j \mid j \notin W^*\} \cup \{p_{max}\})$.
This ensures the payment is at least the winner's bid ($q_i c_i$) and reflects the opportunity cost imposed on the market.

Algorithm 4 shows the implementation used in the simulations based on a price threshold.

Algorithm 4 VCG Auction Simulation (Price Threshold)

Input: Set of sellers N, costs c_i, quantities q_i for each $i \in N$, Buyer's max price p_{max}.
Output: Set of winners W^*, Payments $P = \{P_i\}_{i \in W^*}$, Total Surplus TS.

Initialize $W^* = \emptyset$, $P = \emptyset$.
Identify potential winners $W_{pot} = \{i \in N \mid c_i \leq p_{max}\}$.
if $W_{pot} = \emptyset$ **then**
    return W^*, P, 0.
end if
Identify excluded sellers $N_{excl} = \{j \in N \mid c_j > p_{max}\}$.
if $N_{excl} = \emptyset$ **then**
    $c_{critical} \leftarrow p_{max}$.
else
    $c_{critical} \leftarrow \min\{c_j \mid j \in N_{excl}\}$.
end if
$W^* \leftarrow W_{pot}$.
$TotalPayments \leftarrow 0$.
$TotalTrueCostWinners \leftarrow 0$.
for each winner $i \in W^*$ **do**
    $P_i \leftarrow q_i \times c_{critical}$. ▷ Calculate payment based on critical cost
    $P[i] \leftarrow P_i$.
    $TotalPayments \leftarrow TotalPayments + P_i$.
    $TotalTrueCostWinners \leftarrow TotalTrueCostWinners + q_i \times c_i$.
end for
$TotalBuyerValue \leftarrow p_{max} \times \sum_{i \in W^*} q_i$.
$TS \leftarrow TotalBuyerValue - TotalTrueCostWinners$. ▷ Total Surplus
return W^*, P, TS.

Properties of VCG:
Theorem 3.4 (VCG Properties [23, 22])**.** *The VCG mechanism satisfies the following properties:*

1. ***Surplus Maximization (Efficiency):*** *The VCG mechanism selects the allocation that maximizes the total social surplus.*
2. ***Incentive Compatibility:*** *Bidding truthfully (i.e., bidding c_i if c_i is the true cost) is a dominant strategy for each seller.*
3. ***Individual Rationality (IR):*** *Winning bidders always receive a payment P_i that is greater than or equal to their declared cost valuation $q_i c_i$ (assuming truthful bidding).*
$$P_i \geq q_i c_i$$

Our analysis specifically contrasts the IR property (vs. cost) of VCG with the IR property (vs. standalone payoff r_i) needed for voluntary participation in the broader VCM scheme, using Shapley value as the alternative allocation method.

3.4 Utility Functions

To potentially model farmer preferences beyond simple profit maximization, especially regarding risk, we consider different utility functions $u_i(x)$ mapping monetary payoff x to utility.

- **Linear Utility:** $u_i(x) = x$. This represents risk-neutral behavior.
- **Logarithmic Utility:** $u_i(x) = a_i \log(x) + b_i$ for $x > 0$. This represents risk-averse behavior.

While our core analysis focuses on direct monetary payoffs (linear utility implicitly), considering alternative utility functions can provide insights into how risk aversion might affect coalition stability or mechanism preferences, although this was explored only briefly in the parameter search experiments.

3.5 Evaluation Metrics

We assess the performance and characteristics of these mechanisms using the following metrics:

- **Individual Rationality (IR):** A mechanism satisfies IR if every participating farmer `i` receives a payoff p_i that is weakly preferred to their standalone option payoff.
- **Average Farmer Profit:** The mean payoff $\frac{1}{n}\sum_{i=1}^{n} p_i$ across all farmers considered (n), providing a measure of overall farmer welfare generated by the mechanism.
- **Fairness (Gini Coefficient):** Calculated on the distribution of payoffs $\{p_1, ..., p_n\}$.
  A value of 0 indicates perfect equality (all farmers receive the same payoff), while a value closer to 1 indicates high inequality.
- **Buyer Cost:** The total amount paid by the buyer to all winning farmers in the VCG and Uniform Price auctions, $\sum_{i \in W} p_i$.
- **Total Surplus (VCG):** Measures the total value created, often defined as Total Buyer Value $-$ Total True Cost of Winners.
  For VCG, assuming truthful reporting and a buyer value of P_{max} per credit, this is approximated by $\sum_{i \in W}(P_{max} - c_i)q_i$.
- **Stability (Shapley):** Assessed by checking if the Shapley value allocation $\phi(v)$ lies within the **Core** of the cooperative game (N, v). Only feasible for small N ($N \leq 15$ in our setup).
- **Participation Rate:** The number or percentage of farmers who are selected as winners in the auction mechanisms.

4 Experimental Setup and Methodology

To rigorously evaluate the performance of cooperative game-theoretic allocations and market mechanisms within the context of the proposed Indian agricultural Voluntary Carbon Market (VCM), a series of computational experiments were designed and executed. This section provides a detailed account of the methodology, including the generation of synthetic farmer data, the specific experimental scenarios investigated, the parameter configurations explored, the algorithmic implementations, and the metrics employed for evaluation. The aim is to simulate key aspects of the VCM, particularly focusing on the challenges and opportunities for smallholder farmer participation through Farmer Producer Organizations (FPOs).

4.1 Synthetic Data Generation: Simulating Farmer Heterogeneity

Recognizing the challenges in obtaining comprehensive real-world data at this stage, we generated a synthetic dataset representing a population of Indian farmers. This controlled approach allows for systematic variation of parameters and evaluation of models under different conditions.

Problem Setting and Assumptions: The data generation process simulates n farmers, each characterized by attributes relevant to their potential participation in a VCM and their baseline agricultural activities. Key assumptions include:

- Farmer heterogeneity is crucial and is modeled across multiple dimensions (farm size, costs, potential credits, baseline income).
- Farmers possess private information regarding their true costs (c_i) of generating carbon credits and their baseline standalone payoff (r_i).
- Farmers make decisions (e.g., whether to join an FPO, how to bid in an auction) based on maximizing their expected payoff or utility. The primary analysis assumes risk neutrality (linear utility) .
- The potential quantity of carbon credits (q_i) per farmer is generated based on farm characteristics but is assumed fixed for a given participation decision in the simulation timeframe.

Data Attributes and Distributions: The synthetic dataset was generated using Python scripts leveraging the `numpy.random` library, ensuring reproducibility through a fixed seed (`RANDOM_SEED = 24004`). For a simulated population of $N_{total} = 250$ farmers, the following key attributes were generated for each farmer i:

- `Farmer_ID`**:** Unique string identifier (e.g., 'F00001').
- `Farm_Size_ha`**:** Landholding size in hectares, sampled from a Gamma distribution ($\Gamma(shape = 2, scale = 1.5)$) to reflect the prevalence of smaller farms.
- `Potential_Carbon_Credits_tCO2e` **(q_i):** The potential number of credits a farmer can generate. Sampled from $\Gamma(shape = 2.5, scale = 1.8)$, clipped below at 0.1 tCO2e to ensure positivity.
- `Standalone_Payoff_INR` **(r_i):** The farmer's baseline annual income from conventional farming. Sampled from a Normal distribution ($\mathcal{N}(\mu = 20000, \sigma = 5000)$), rounded to the nearest 100 INR, and clipped below at 5000 INR.
- `True_Cost_per_Credit_INR` **(c_i):** The farmer's private marginal cost to generate one carbon credit. Sampled from $\Gamma(shape = 3, scale = 800)$, with a base of 500 INR added, clipped below at 100 INR. This cost represents the additional expenses or effort required for sustainable practices, normalized by the credits generated.
- `Certification_Cost_Individual_INR`**:** A simulated cost if the farmer were to undergo certification individually, sampled uniformly from $[4000, 9000]$ INR.
- `Risk_Aversion_Coeff`**:** A coefficient sampled from a Beta distribution ($Beta(2, 3)$, scaled and shifted to $[0.1, 5.1]$) to represent potential variations in risk attitudes, available for future analysis using non-linear utility functions.
- `Other Categorical/Demographic Attributes`: Variables like `Land_Tenure_Type`, `Education_Level`, `Gender`, `Debt_Status`, `Crop_Type`, `Market_Access`, `Previous_Coalition_Experience`, and `Farm_Location_State` were sampled using `numpy.random.choice` with probability distributions based on general Indian agricultural contexts.

This generated dataset, comprising 250 farmer profiles, forms the basis for all subsequent experiments. For simulations requiring fewer farmers ($n < 250$), subsets were randomly sampled from this base dataset.

4.2 Cooperative Game Analysis: FPO Formation and Allocation

To investigate the benefits and internal dynamics of farmers collaborating within FPOs, we modeled the situation as a cooperative TU game (N, v) as defined in Section 3.

Characteristic Function Implementation: The value $v(S)$ for a coalition S was calculated using Equation 1: $v(S) = \alpha \sum_{i \in S} r_i + \beta(\sum_{i \in S} r_i)^2$. The implementation involved retrieving the `Standalone_Payoff_INR` (r_i) for each farmer $i \in S$ from the dataset and applying the formula based on the specified α and β parameters for that simulation run. This function captures both the baseline contribution scaled by α and the synergy effect scaled by β.

Shapley Value Computation: The Shapley value $\phi_i(v)$ for each farmer $i \in N$ was computed to determine a fair allocation of the grand coalition's value $v(N)$.

- *Exact Method:* For simulations with a small number of farmers ($n \leq 10$), the exact Shapley value was computed using the permutation-based approach outlined in Algorithm 2. This involves iterating through all $n!$ permutations, calculating marginal contributions for each player in each permutation, and averaging.
- *Monte Carlo Approximation:* For larger n ($n > 10$), exact computation becomes infeasible. We employed Monte Carlo sampling as described in Algorithm 3. A substantial number of random permutations (M, typically set to 10,000) were generated, and the expected marginal contribution for each player was estimated by averaging over these samples.

The computed Shapley values $\{\phi_i\}_{i \in N}$ represent the payoff allocated to each farmer under this fairness principle.

Core Stability Analysis: To assess whether the Shapley value allocation leads to stable FPOs, we checked if the allocation vector x belonged to the Core. Due to the computational complexity ($O(2^n \times n)$), the Core check (Algorithm 1) was performed only for simulations with $n \leq 15$.

Experiments Conducted: Specific experiments focused on

1. **Individual vs. Grand Coalition (N=12):** Compared the Shapley value payoff ϕ_i for each farmer in the grand coalition ($N = 12$) against their standalone payoff r_i. Calculated average gain and percentage better off.
2. **Coalition Size Effect (N=3, 5, 8, 12):** For various coalition sizes, sampled multiple coalitions randomly, calculated the average value per farmer ($v(S)/|S|$) and the average Shapley value within the coalition. Examined trends with increasing size.
3. **Allocation Rule Stability (N=12):** Checked if Shapley, Equal Split ($x_i = v(N)/n$), and Proportional Split ($x_i = (r_i / \sum r_j) \times v(N)$) allocations were Core-stable.
4. **Parameter Sensitivity (N=100):** Varied α and β systematically and computed the resulting average Shapley payoffs and assessed Individual Rationality ($x_i \geq r_i$).

4.3 Mechanism Design Analysis: Auction Simulations

To evaluate market mechanisms for trading carbon credits generated by potentially heterogeneous farmers, we simulated auction scenarios.

A. VCG Auction Implementation: The VCG mechanism was implemented based on the theoretical description in Section 3 and Algorithm 4.

- *Input:* Set of participating farmers N, their true costs per credit c_i, potential credit quantities q_i, and a market clearing condition, typically modeled as a maximum price p_{max} the buyer is willing to pay per credit.
- *Assumption:* Farmers bid truthfully (i.e., bid their true cost c_i), consistent with the strategy-proof property of VCG.
- *Allocation:* Winners (W^*) are determined as all farmers i for whom $c_i \leq p_{max}$.
- *Payment Calculation:* The critical cost $c_{critical}$ is determined as the minimum cost among losing bidders ($j \notin W^*$) or p_{max} if all bidders with $c_i \leq p_{max}$ win. Each winner $i \in W^*$ receives a payment $P_i = q_i \times c_{critical}$.
- *Surplus Calculation:* Total social surplus is calculated as $\sum_{i \in W^*} q_i(p_{max} - c_i)$.

B. Uniform Price Auction Implementation: A simple ascending-price auction was simulated for comparison, as

- *Input:* Sellers N, costs c_i, quantities q_i, and a total buyer demand quantity Q_{demand}.
- *Output:* The set of winners W^*, the clearing price p_{clear}, and the payments P_i for each winner.

Experiments Conducted:

1. **Supply Curve Generation (N=250):** Ran the VCG auction simulation across a range of 'vcg_price_per_credit' values (500 to 4000 INR). Recorded the total credits supplied ($\sum q_i$ for winners) at each price point.
2. **VCG Outcome Analysis (N=250):** For the same price range as above, calculated and tracked key VCG metrics: number of winners, total surplus, total payments, Gini coefficient of payments among winners, IR status (vs. cost c_i), and budget balance.
3. **Mechanism Comparison (N=12 to 250):** Compared VCG and Uniform Price auctions against the Shapley allocation (from the cooperative game model, using $\alpha = 1.0, \beta = 0.0$ for a baseline comparison) across a varying 'market_price_per_credit'. Focused on comparing average farmer profit, Gini coefficient, and IR percentage (vs. standalone payoff r_i).

4.4 Evaluation Metrics Implementation

The metrics defined in Section 3.5 were implemented computationally:

- **Gini Coefficient:** Calculated using the standard formula involving the sum of absolute differences between all pairs of payoffs, normalized by the total payoff and number of players.
- **Individual Rationality:** Implemented as direct comparisons using a small tolerance ϵ for floating-point precision. For cooperative settings, $x_i \geq r_i - \epsilon$. For VCG, $P_i \geq (q_i c_i) - \epsilon$ for $i \in W^*$. Percentage IR was calculated as $\frac{\text{number of farmers meeting IR}}{\text{total number of relevant farmers}} * 100$.
- **Core Check and VCG :** Implemented as per Algorithm 1 and Calculated directly from the outputs of the VCG simulation (Algorithm 4) respectively.

These metrics allowed for a quantitative comparison of the different scenarios, parameters, and mechanisms explored in the study. The codebase includes functions dedicated to these calculations. The complete codebase for the experiments and simulations is available on GitHub at the following link: *https://github.com/Mahanth-Maha/GameTheory2025MiniProject*.

5 Results and Discussion

This section presents the findings from the computational experiments detailed in Section 4. We analyze the results concerning coalition formation, stability, market mechanism performance, and parameter sensitivity, interpreting their implications for the design of a Voluntary Carbon Market (VCM) in the Indian agricultural sector.

5.1 Benefits and Stability of FPO Coalitions

A primary objective was to assess the potential benefits of farmers forming coalitions (FPOs) and the stability of different methods for allocating the generated value, using the cooperative game model defined in Section 4.2.

Value Generation and Economies of Scale: Simulations with varying coalition sizes ($N \in \{3, 5, 8, 12\}$) were conducted using baseline parameters ($\alpha = 1.0, \beta = 0.01$) to investigate the effect of aggregation. Figure 1 displays the average value per farmer ($v(S)/|S|$) and the average Shapley value per farmer within the coalition as a function of coalition size $|S|$.

As clearly shown in Figure 1 (Left), the average value generated per farmer increases substantially with the size of the coalition. This trend is mirrored in the average Shapley value allocated per farmer (Figure 1, Right). This result strongly supports the hypothesis that aggregation through FPOs can yield significant benefits.

Furthermore, comparing the Shapley value payoff (ϕ_i) with the standalone payoff (r_i) for a specific coalition of N=12, we found that all farmers (100%) received a significantly higher payoff through the coalition under these parameters ($\alpha = 1.0, \beta = 0.01$). The average gain per farmer ($\phi_i - r_i$) was substantial, indicating a strong economic incentive to join the FPO, and the value is distributed fairly (as approximated by Shapley).

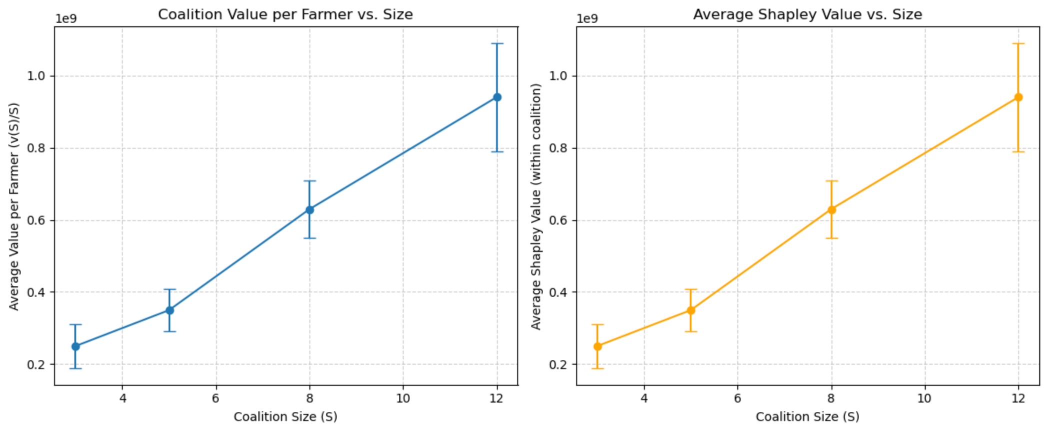

Figure 1: Analysis of coalition value and Shapley value distribution as a function of coalition size (N=3, 5, 8, 12). Left: Average value per farmer ($v(S)/|S|$). Right: Average Shapley value per farmer ($\phi_i(v)$). Error bars indicate standard deviation across simulation runs with different sampled coalitions.

Core Stability of Allocations: For the N=12 simulation, we investigated the stability of different allocation rules by checking if they reside within the Core (Algorithm 1). The results, summarized in Table 1, indicate that under the baseline parameters ($\alpha = 1.0, \beta = 0.0$), all tested allocation rules were found to be Core-stable.

Table 1: Core Stability Analysis for N=12 Simulation (Baseline Parameters)

Allocation Rule	Is in Core?	Gini Coefficient
Shapley Value	True	≈ 0.1195
Equal Split	True	0.0000
Proportional (Standalone)	True	≈ 0.1205

Note: Results extracted from simulation logs for N=12, $\alpha = 1.25, \beta = 0.0$. Gini coefficients provide a measure of fairness.

This finding is significant: it suggests that for smaller FPOs and under conditions where cooperation is beneficial ($\beta > 0$), mechanisms like the Shapley value not only provide fair distribution (low Gini, comparable to proportional split based on baseline r_i) but also lead to stable outcomes where no subgroup has an incentive to deviate. The Equal Split method, while perfectly egalitarian (Gini=0), was also stable in this instance but might be less reflective of differing contributions in practice. The Core stability check was computationally infeasible for larger N (e.g., N=100, N=250) in our experiments, highlighting a limitation for verifying stability in larger, more realistic FPO settings without using approximation techniques or focusing on specific coalition structures.

5.2 Impact of Farmer Heterogeneity on Coalition Benefits

Real-world FPOs often consist of members with diverse characteristics, notably varying farm sizes and baseline incomes (r_i). A concern might be whether larger, potentially more influential farmers capture a disproportionate share of coalition benefits, leaving small farmers with little incentive to join. To investigate this, we conducted specific simulations including distinct groups of 'Small' (lower r_i, lower q_i) and 'Large' (higher r_i, higher q_i) farmers within the same coalition (N=15: 10 Small, 5 Large). We analyzed the Shapley value allocation under different parameter settings (α, β).

Scenario 1: No Synergy, No Scaling ($\alpha = 1.0, \beta = 0.0$). As theoretically expected, when no additional value is generated beyond the sum of standalone payoffs, the Shapley value returns exactly the standalone payoff to each farmer ($\phi_i = r_i$). In this case, neither small nor large farmers have a strict economic incentive to join, although they are not made worse off.

Scenario 2: Baseline Scaling, No Synergy ($\alpha = 1.25, \beta = 0.0$). In the absence of synergy but with a positive scaling factor ($\alpha > 1$), all farmers received a Shapley value exactly 25% higher than their standalone payoff ($\phi_i = 1.25 \times r_i$). Figure 2 illustrates the average absolute and percentage gains.

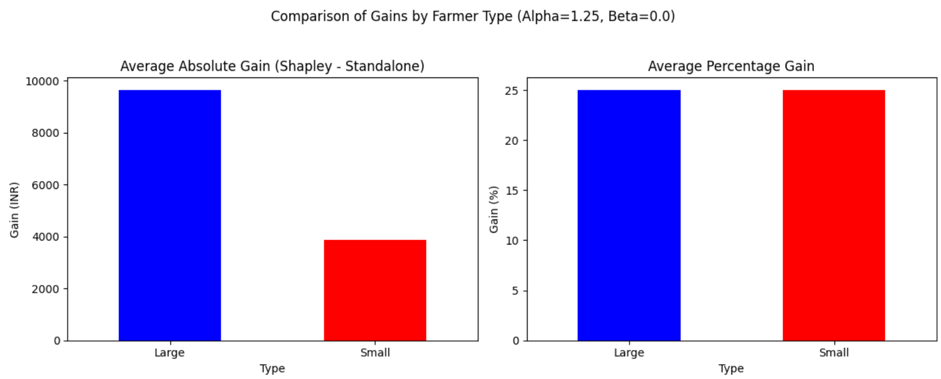

Figure 2: Average Absolute and Percentage Gains by Farmer Type (N=15, $\alpha = 1.25, \beta = 0.0$). Shows Shapley Gain over Standalone Payoff.

Figure 2 clearly shows that while large farmers gain more in absolute INR terms, the percentage gain is identical (25%) for both groups. This demonstrates that the baseline scaling factor α acts uniformly relative to initial payoffs under the Shapley allocation in this model. Again, this provides a clear, equitable (in percentage terms) incentive for both small and large farmers to participate.

5.3 Performance of Market Mechanisms

We analyzed the performance of the VCG auction mechanism for trading carbon credits and compared it against the Shapley allocation and a simple Uniform Price auction.

VCG Auction Dynamics: Simulations using N=250 farmers and varying the buyer's maximum willingness-to-pay (`market_price_per_credit`) allowed us to trace market dynamics under the VCG mechanism (Algorithm 4). Figure 3 shows the resulting VCM supply curve.

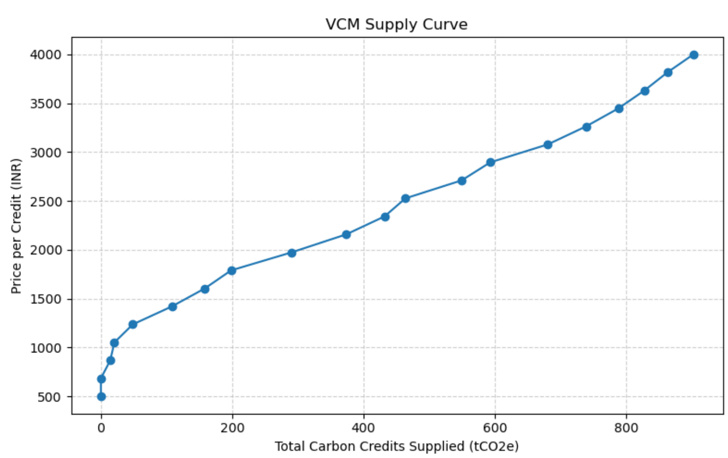

Figure 3: Simulated Voluntary Carbon Market (VCM) Supply Curve using VCG Auction (N=250). Shows total credits supplied vs. market price per credit.

The supply curve exhibits the expected upward slope, as the price offered per credit increases, more farmers find it profitable to participate (i.e., their true cost c_i is below the price), leading to a greater aggregate supply of credits.

Figure 4 provides a more detailed view of VCG outcomes as a function of price.

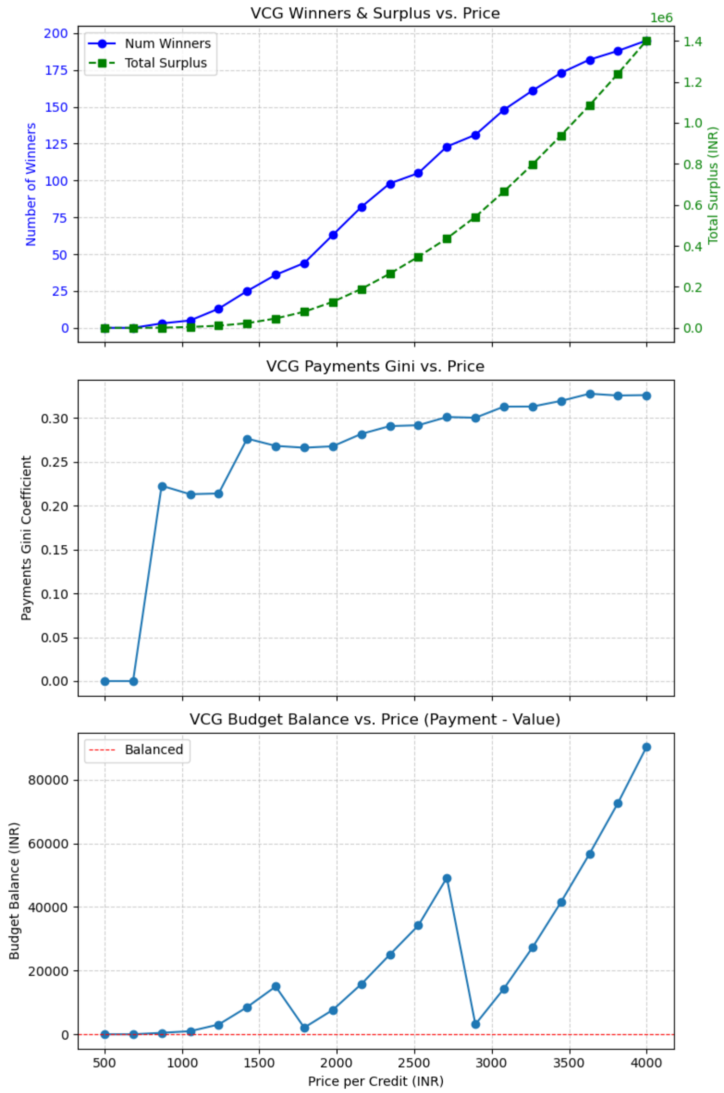

Figure 4: VCG Auction Outcomes vs. Price per Credit (N=250). Top: Number of winning farmers and total social surplus. Middle: Gini coefficient of payments among winners. Bottom: VCG budget balance (Total Payments - Total Buyer Value at p_{max}).

Key observations from Figure 4:

- **Efficiency and Participation:** Both the number of winning farmers and the total social surplus increase steadily with the market price (Top panel), demonstrating the efficiency property of VCG in maximizing value capture as prices rise.
- **Payment Fairness (Winners):** The Gini coefficient among winning farmers (Middle panel) fluctuates but remains in a moderate range (mostly 0.25-0.32)suggesting that VCG itself doesn't induce extreme
- **Budget Balance:** The VCG mechanism consistently generates a budget surplus for the "auctioneer" or market platform (Bottom panel), as total payments (based on critical costs) are generally less than the total value perceived by the buyer at the maximum price (p_{max}). This surplus increases with price, representing value not captured by participating farmers or the buyer directly. This could potentially fund the platform but might be perceived as extracting value.

- **IR vs. Cost:** Simulations confirmed that the VCG mechanism always satisfied Individual Rationality with respect to the farmer'`strue cost` c_i for all winners (i.e., $P_i \geq q_i c_i$), as expected theoretically.

Mechanism Comparison–Profitability, Fairness, and Participation A crucial set of experiments compared Shapley Allocation (representing cooperative distribution within an FPO), VCG Auction, and Uniform Price Auction across a range of market prices. The results are presented in the Figures 5, 6, and 7.

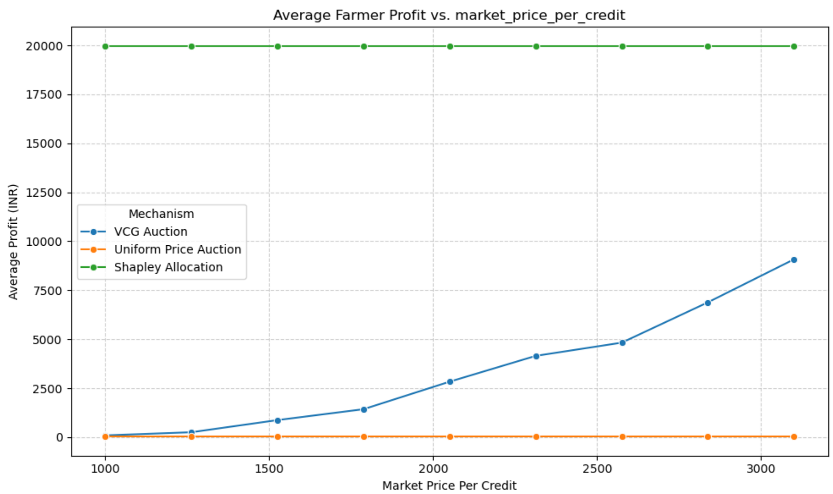

Figure 5: Average Farmer Profit vs. Market Price per Credit for Different Mechanisms (N=250, $\alpha = 1.0, \beta = 0.0$).

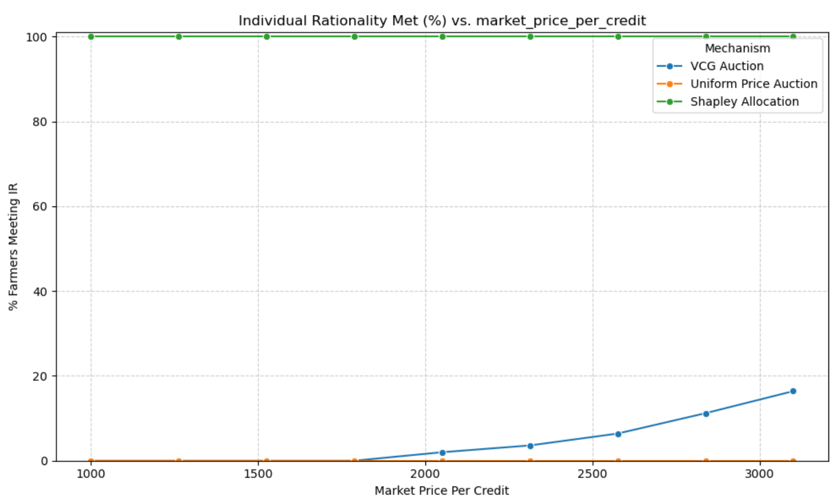

Figure 6: Individual Rationality Met (%) vs. Market Price per Credit (N=250, $\alpha = 1.0, \beta = 0.0$). IR is checked against the farmer's standalone payoff r_i.

The comparison of mechanisms reveals several important insights

- **Profitability (Fig. 5):** Shapley Allocation (with $\alpha = 1.0, \beta = 0.0$) yields a constant average profit equal to the average standalone payoff r_i, as expected when there's no synergy. VCG profit increases with price but remains significantly lower than Shapley in this baseline case. Uniform Price yields negligible average profit. When synergy is introduced ($\beta > 0$, see sensitivity analysis below in the next sub section 5.4), Shapley profit dramatically increases.

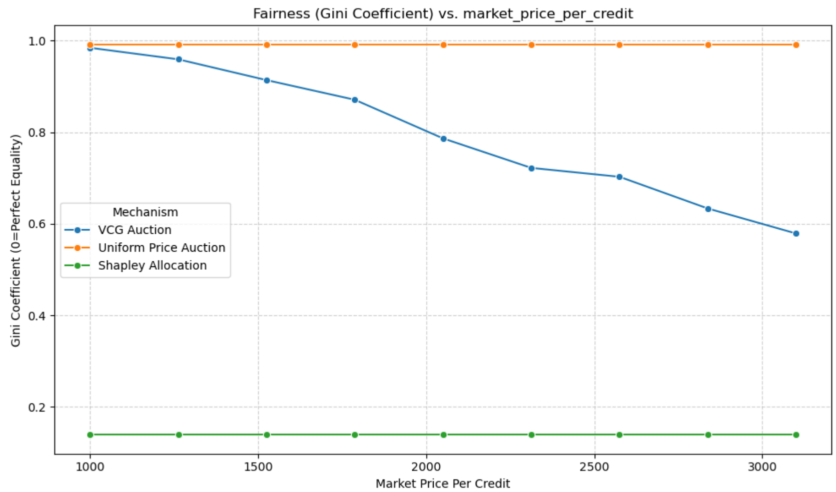

Figure 7: Fairness (Gini Coefficient) vs. Market Price per Credit (N=250, $\alpha = 1.0, \beta = 0.0$).

- **Individual Rationality vs. Standalone Payoff (Fig. 6):** This is the most critical result for a *voluntary* market. Shapley Allocation (with $\alpha = 1.0, \beta = 0.0$) achieves 100% IR, meaning no farmer is worse off than their baseline r_i. In stark contrast, VCG shows very poor IR performance, only exceeding 10% at the highest prices. Uniform Price shows 0% IR. This suggests that without modification or significant synergy capture, auction mechanisms like VCG may fail to incentivize broad voluntary participation if farmers compare the outcome to simply continuing their traditional farming.
- **Fairness (Fig. 7):** Shapley Allocation demonstrates excellent fairness with a low and stable Gini coefficient (≈ 0.14). VCG fairness among participants varies significantly with price but is generally much worse (higher Gini). Uniform Price Auction shows near-perfect inequality (Gini ≈ 1) because only a few low-cost farmers win and get paid the (low) clearing price, while most get zero.

These results strongly favor the Shapley value approach from a farmer participation and equity perspective, especially when compared against baseline farming activities (r_i).

5.4 Parameter Sensitivity–Impact of α and β

Given the importance of the characteristic function parameters, we analyzed their impact on Shapley payoffs for N=100 farmers.

Impact of α (Baseline Scaling, $\beta = 0$). Simulations systematically varying α from 0.75 to 1.50 (while $\beta = 0$) showed a direct linear relationship between α and the average Shapley payoff ϕ_i.

- When $\alpha < 1.0$ (e.g., $\alpha = 0.75$), $\phi_i < r_i$ for all farmers. This resulted in 0% IR satisfaction, indicating that scaling down baseline contributions makes cooperation unattractive.
- When $\alpha = 1.0$, $\phi_i = r_i$ for all farmers (Gain=0). This satisfies IR exactly but offers no strict incentive over standalone farming.
- When $\alpha > 1.0$ (e.g., $\alpha = 1.05, 1.10, \ldots, 1.50$), $\phi_i > r_i$ for all farmers, satisfying IR and providing a positive gain that increases linearly with α.

This highlights that ensuring $\alpha \geq 1.0$ is crucial if there are no additional synergy benefits ($\beta = 0$).

5.5 Discussion of Results and Implications

The experimental findings provide several key insights relevant to designing and implementing the proposed VCM for Indian agriculture:

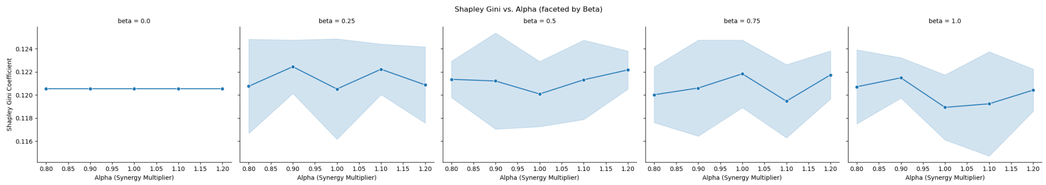

Figure 8: Shapley Gini Coefficient vs. α, faceted by β (N=12).

1. **Value of Aggregation (FPOs):** The simulations strongly support the government's emphasis on farmer aggregation [17]. Collective action through FPOs demonstrably increases the potential value generated per farmer (Figure 1).
2. **Shapley Value as a Strong Allocation Candidate:** The Shapley value emerges as a compelling mechanism for distributing collective VCM revenue within FPOs.
   - It offers demonstrably *fair* outcomes (low Gini, Figures 7, 8, Table 1).
   - Crucially, under reasonable parameter assumptions ($\alpha \geq 1$ or $\beta \geq 0$), it satisfies *Individual Rationality* compared to standalone farming (Figure 6, sensitivity analysis), which is paramount for ensuring *voluntary* participation.
   - For smaller coalitions (N=12), it resulted in *Core-stable* allocations (Table 1), suggesting it can foster stable cooperation.
3. **Limitations of Standard Auction Mechanisms (for IR):** While VCG auctions are efficient at maximizing social surplus (Figure 4) and incentivize truthful cost revelation, they perform poorly when farmer participation depends on exceeding their standalone farming payoff r_i, rather than just covering their abatement cost c_i. The low IR percentages observed for VCG and Uniform Price auctions (Figure 6) suggest they might not be suitable as the sole mechanism driving participation in a purely voluntary setting without further incentives or modifications.
4. **Need for Integrated Approach:** The findings suggest an integrated approach is needed. FPOs are vital for aggregation and potentially realizing synergies (modeled via cooperative game theory). The mechanism for distributing benefits within the FPO (e.g., Shapley) must ensure fairness and meet participation constraints. The mechanism for selling credits to the market (e.g., auctions) needs to be designed considering its impact back on farmer incentives and overall surplus distribution. Perhaps hybrid mechanisms combining auction price discovery with cooperative surplus distribution warrant further investigation.
5. **Limitations and Future Directions:** The reliance on synthetic data, the specific functional form chosen for $v(S)$, and the computational limits on Core analysis represent limitations. Future work should involve calibration with real-world FPO data, exploring alternative characteristic functions (e.g., incorporating costs explicitly), investigating Core approximations for larger N, and designing/analyzing novel hybrid market mechanisms.

In summary, the experiments confirm the potential of FPO-based VCMs but highlights the critical importance of mechanism design choices, particularly regarding revenue allocation, for ensuring farmer participation, fairness, and stability. The Shapley value demonstrates strong potential, while standard auctions like VCG require careful consideration regarding their suitability for incentivising voluntary smallholder engagement.

6 Conclusion and Future Work

India's initiative to establish a Voluntary Carbon Market (VCM) for its agriculture sector holds significant potential for promoting sustainable practices, enhancing farmer livelihoods, and contributing to national climate goals [17]. However, the success of this initiative, particularly in engaging the vast majority of small and marginal farmers, depends critically on addressing inherent challenges related to transaction costs, fair value distribution, and participation incentives. This paper employed cooperative game theory and mechanism design principles to analyze these challenges and evaluate potential solutions.

Our analysis, supported by computational simulations, yields several key conclusions (NOTE : These results are purely based on the Synthetic Data experiments, which may not align with the actual real-world data, hence treat these results as indicative rather than conclusive).

1. **Essential Role of FPOs:** Farmer aggregation through mechanisms like Farmer Producer Organizations (FPOs) is not merely beneficial but likely essential for VCM viability for smallholders. Significant potential for value creation exists through coalition formation, particularly when synergistic effects or economies of scale are realized (Section 5, Figure 1).
2. **Shapley Value for Fair Allocation:** The selection of an appropriate mechanism to allocate the collective value generated within FPOs is critical. The Shapley value consistently emerged as a strong candidate, providing:
   - Equitable distributions (low Gini coefficient).
   - Satisfaction of individual rationality (IR) compared to baseline standalone farming payoffs under assumed conditions (Section 5, Figures 6, 7).

   This alignment with farmer participation constraints is vital for a *voluntary* market.
3. **Challenges with Auction Mechanisms:** While standard auction mechanisms like VCG offer efficiency properties in theory and practice (Section 5, Figure 4), simulations highlight:
   - Potential shortcomings in guaranteeing participation incentives relative to farmers' existing agricultural options.
   - The need for complementary mechanisms or modifications in this specific context.
4. **Sensitivity to Cooperation Benefits:** Outcomes are highly sensitive to how the benefits of cooperation (parameters α and β in our model) are estimated and realized, emphasizing the need for careful, empirically grounded modeling.

Practically, this research underscores the utility of game theory and mechanism design as indispensable tools for VCM design. They provide a structured framework to move beyond high-level policy goals [17] towards implementable rules and structures. Analyzing FPOs as cooperative games allows for predicting stability (using the Core, Section 4.2) and designing fair internal allocation rules (like Shapley). Applying mechanism design helps evaluate external market interactions (like auctions, Section 4.3) based on incentive compatibility, efficiency, and participation constraints. The quantitative comparisons offered here provide evidence-based insights for policymakers and FPO managers navigating these complex design choices.

Future Work : This study opens several avenues for future research, few of which are

1. **Empirical Validation:** Calibrating the characteristic function ($v(S)$) and farmer cost/payoff parameters using real-world data from Indian FPOs and agricultural settings is a critical next step to enhance model realism.
2. **Aggregator as a Strategic Player:** A promising direction involves modeling the FPO or a third-party aggregator not just as a facilitator but as a strategic player in the game. This requires defining the aggregator's objectives (e.g., maximizing member welfare, maximizing own profit, ensuring stability) and analyzing its optimal strategies for recruiting members, managing resources, and interacting with the carbon market. An improved coalition value function, potentially incorporating aggregator costs and value-add, could be developed, as partially outlined in the Appendix (Section A).

Addressing these questions will further refine our understanding and contribute to the successful implementation of an effective Voluntary Carbon Market benefitting India's farmers and environment.

Acknowledgements

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Use of AI :

In the process of preparing this document, the authors have utilized various AI language models and assistants, such as GPT (ChatGPT) and BERT (Gemini), to enhance the writing quality, provide summaries of source materials and codes, and gain additional insights. However, the authors diligently verified all information provided by these AI tools to ensure accuracy and factual integrity.

Furthermore, specific assistance was leveraged from by Large Language Models (LLMs) during the data generation phase. LLMs were used to explore and select statistical distributions (such as Gamma, Normal, Beta) and their parameter ranges for generating the synthetic farmer dataset described in Section 4.1. This facilitated the creation of a heterogeneous dataset representing key farmer attributes relevant to the Voluntary Carbon Market context especially for the Indian agriculture sector.

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A Analysis of an Aggregator-Mediated VCM Model

The main analysis in this report models Farmer Producer Organizations (FPOs) as cooperative coalitions where the total generated revenue is distributed among farmers. While in reality the *FPOs* or *third-party aggregators* often act as intermediaries with their **own costs and potential profit motives**. This appendix presents an extension by explicitly modeling such an aggregator, analyzing its impact on farmer payoffs, participation incentives, and overall market dynamics.

A.1 Model Setting

We extend the cooperative game framework (N, v) to include a central aggregator entity A.

Definition A.1 (Aggregator included Game setting)**.** *The components of the aggregator included model are*

- ***Farmers:*** $N = \{1, 2, \ldots, n\}$*, each with baseline standalone payoff* r_i*.*
- ***Aggregator:*** A*, supporting coalition* $S \subseteq N$*.*
- ***Gross Value (***$V(S)$***):*** *The underlying value generated by coalition* S *before considering aggregator costs or share, modeled as:*

$$V(S) = \alpha \sum_{i \in S} r_i + \beta \left(\sum_{i \in S} r_i \right)^2$$

  where $\alpha, \beta \geq 0$ *are base potential parameters (same as previous).*
- ***Aggregator Base-Pay Cost Function (***$C_A(S)$***):*** *Costs incurred by* A *for managing* S *, which may depend on the number of farmers in* S *and other factors. A simple linear model which considers the coalition size* $|S|$ *can be*

$$C_A(S) = C_{base} + C_{var} \times |S|$$

  with $C_{base} \geq 0$ *(base pay/fixed overhead),* $C_{var} \geq 0$ *(variable cost per farmer), and (for* $S \neq \emptyset, C_A(\emptyset) = 0$*).*
- ***Net Available Value (***$V_{net}(S)$***):*** *Value remaining after costs incurred by the aggregator*

$$V_{net}(S) = \max(0, V(S) - C_A(S))$$

- ***Aggregator Commission Rate (***δ***):*** *The fraction* $\delta \in [0, 1)$ *of the net available value that the aggregator retains as profit. This is a strategic decision can motivate the aggregator's behavior to maximize its own utility by maximizing the net value.*
- ***Aggregator Profit (***$\pi_A(S)$***):*** *The aggregator's retained share*

$$\pi_A(S) = \delta \times V_{net}(S)$$

- ***Net Value for Farmers** ($V_F(S)$): The value remaining for distribution among farmers in S*

$$V_F(S) = V_{net}(S) - \pi_A(S) = (1 - \delta)V_{net}(S) \tag{2}$$

  This V_F serves as the new characteristic function for the farmers cooperative subgame with aggregator as mediator.

- ***Farmer Payoff Allocation** (x_i): Assumed to be determined by the Shapley value applied to the farmers cooperative game (N, v_F),*

$$x_i = \phi_i(v_F)$$

- ***Aggregator Profit:** The aggregator's profit for the grand coalition N is given by*

$$\pi_A(N) = C_A(N) + \delta \times V_{net}(N)$$

- ***Farmer IR Constraint:** Farmer i participates if $x_i \geq r_i$.*

This formulation captures the aggregator's role as a strategic player, influencing the net value available to farmers and their incentives to participate in the coalition. This motivates the aggregator to set its commission rate δ and manage its costs $C_A(S)$ effectively to maximize its profit while ensuring sufficient net value remains for farmers to incentivize their participation.

A.2 Experimental Methodology

We simulated the aggregator's impact using the synthetic dataset ($N_{total} = 250$, generation described in Section 4.1) with varing the aggregator's commission rate δ.

Experimental Parameter Settings :

- Farmer Population (N): Experiments run for $N = 15$ (allowing Core checks) and $N = 250$ (larger scale, hence No Core checks).
- Base Potential: Fixed at $\alpha = 1.25$ and $\beta = 0.0$ for clarity, isolating the effect of costs and commission on a simple linear potential gain (25% gross boost over baseline).
- Aggregator Costs: Set to $C_{base} = 10000$ INR and $C_{var} = 300$ INR/farmer.
- Commission Rate (δ): Varied from 0.0 (0%) to 0.5 (50%) in steps of 0.05.
- Shapley Calculation: Exact method for $N \leq 15$, Monte Carlo for $N > 15$.
- Core Check: Performed for $N = 15$ only, using the net farmer value function $V_F(S)$.

Procedure: For each value of δ:

1. Define the specific net farmer value function $V_F(S)$ based on Equation 2 using the current δ and fixed cost/potential parameters.
2. Compute the Shapley value allocation $x = (\phi_1(v_F), \ldots, \phi_n(v_F))$ using the appropriate algorithm (Exact or MC).
3. Calculate the aggregator's profit $\pi_A(N)$ for the grand coalition.
4. Evaluate metrics:
   - Average farmer payoff.
   - Average absolute and percentage gain ($x_i - r_i$).
   - IR Met Percentage ($x_i \geq r_i$).
   - Gini coefficient of x.
   - Core stability status (for $N = 15$).

The full implementation details are available in the project's codebase[2].

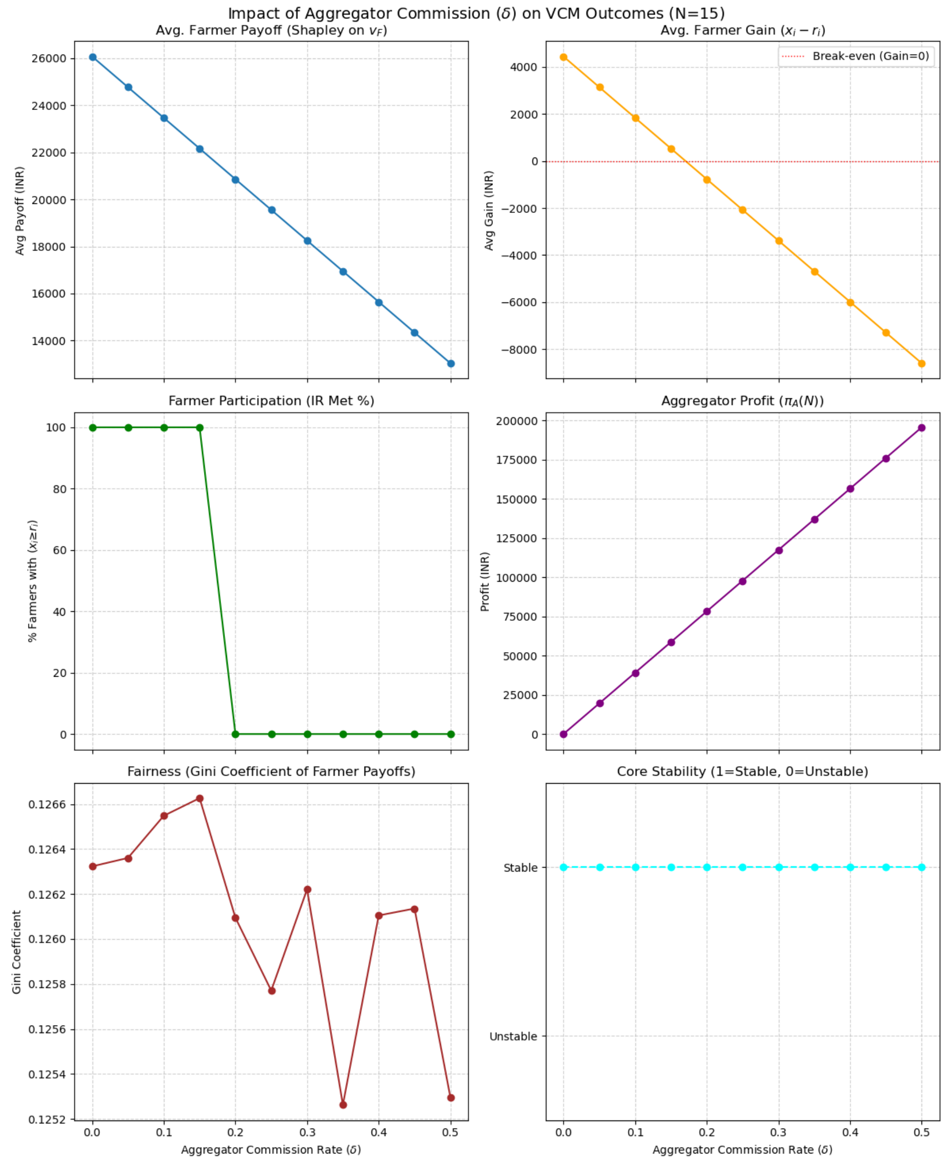

Figure 9: Impact of Aggregator Commission (δ) for N=15.(Base parameters: $\alpha = 1.25, \beta = 0.0, C_{base} = 10000, C_{var} = 300$)

A.3 Results and Discussion

The simulations reveal a clear trade-off between the aggregator's commission rate (δ) and farmer participation. Figures 9 and 10 plot the key metrics against δ for both $N = 15$ and $N = 250$ scenarios.

Key Observations:

- **Farmer Payoffs vs. Commission:** As δ increases, the average farmer payoff decreases linearly. This is expected as the aggregator retains a larger fraction of the net available value.
- **Participation Threshold:** For both $N = 15$ and $N = 250$, the percentage of farmers meeting the IR constraint ($x_i \geq r_i$) drops from 100% to 0% when δ increases from 0.15 to

[2] Code available at: `https://github.com/Mahanth-Maha/GameTheory2025MiniProject`

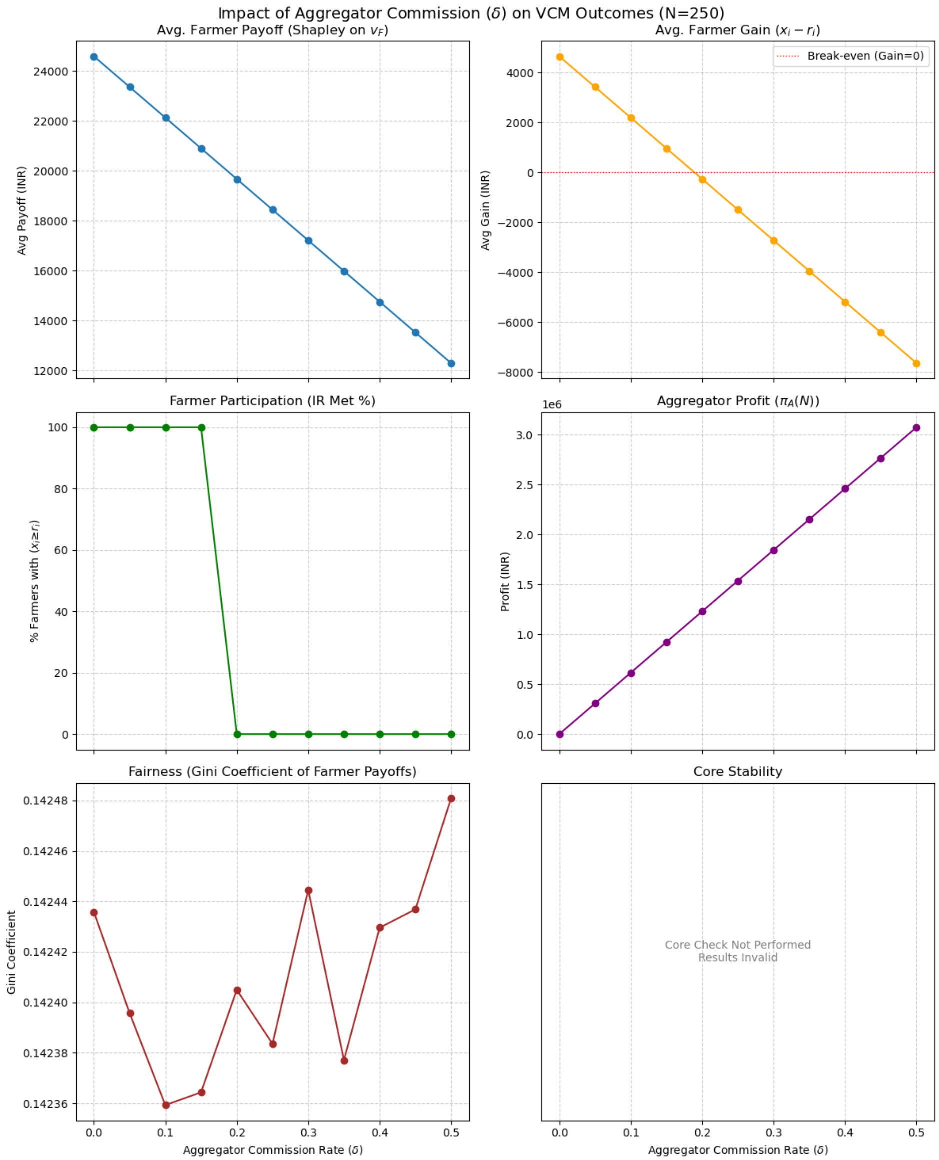

Figure 10: Impact of Aggregator Commission (δ) for N=250. (Base parameters: $\alpha = 1.25, \beta = 0.0, C_{base} = 10000, C_{var} = 300$)

0.20. This implies that if the aggregator takes 20% or more commission *after* covering its operational costs, the remaining value distributed via Shapley is **insufficient to incentivize voluntary participation** compared to standalone farming, under these specific cost and potential parameters ($\alpha_0 = 1.25, \beta_0 = 0.0$).

- **Aggregator Profit:** The aggregator's total profit $\pi_A(N)$ increases linearly with δ, reaching its maximum when farmer participation might already be None.
- **Fairness (Gini):** The Gini coefficient of the farmer payoffs x_i remains consistently low and stable (≈ 0.126 for N=15, ≈ 0.142 for N=250) regardless of the commission rate δ.
- **Core Stability (N=15):** The Shapley allocation remained Core-stable (with respect to the net farmer value function v_F) across the entire range of δ. This means that even when the allocation fails the crucial participation IR constraint ($x_i \geq r_i$), no subgroup of farmers could guarantee doing better by splitting off and only considering the value $v_F(S)$ generated via the aggregator for their subgroup.

A.4 Conclusion of Appendix

This extension explicitly modeling the aggregator provides valuable insights. It demonstrates that aggregator operational costs and commission strategies directly impact the net value available to farmers and, consequently, their incentive to participate in the VCM. A critical threshold exists for the commission rate, beyond which voluntary participation collapses, even if the internal allocation (like Shapley) is fair and theoretically stable.

The stability of the Shapley allocation within the Core (for small N) even at high commission rates, while is practically less relevant if the payoffs fail the initial participation constraint. This shows that VCM design must carefully balance the need for aggregator viability (covering costs $C_A(S)$ and potentially earning a profit $\pi_A(S)$) with the necessity of providing sufficient net revenue back to farmers ($x_i = \phi_i(v_F) \geq r_i$) to ensure their voluntary engagement.

Future work could explore optimal commission structures (δ) or alternative aggregator models (e.g., non-profit FPOs minimizing δ vs. for-profit aggregators) and their impact on overall VCM success [32, 30]. The computational aspects of finding optimal aggregator strategies or analyzing these more complex multi-level games also present interesting challenges [5].